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New constraints on parametrised modified gravity from correlations of the CMB with large scale structure

Tommaso Giannantonio,[1] Matteo Martinelli,[2] Alessandra Silvestri,[3] and Alessandro Melchiorri[2]

[1] *Argelander–Institut für Astronomie der Universität Bonn, Auf dem Hügel 71, D-53121 Bonn, Germany*
[2] *Università degli studi di Roma "La Sapienza", P.le Aldo Moro 5, I-00185 Roma, Italy*
[3] *Kavli Institute for Astrophysics and Space Research, MIT, Cambridge, MA 02139 USA*

We study the effects of modified theories of gravity on the cosmic microwave background (CMB) anisotropies power spectrum, and in particular on its large scales, where the integrated Sachs–Wolfe (ISW) effect is important. Starting with a general parametrisation, we then specialise to $f(R)$ theories and theories with Yukawa–type interactions between dark matter particles. In these models, the evolution of the metric potentials is altered, and the contribution to the ISW effect can differ significantly from that in the standard model of cosmology. We proceed to compare these predictions with observational data for the CMB and the ISW, performing a full Monte Carlo Markov chain (MCMC) analysis. In the case of $f(R)$ theories, the result is an upper limit on the lengthscale associated to the extra scalar degree of freedom characterising these theories. With the addition of data from the Hubble diagram of Type Ia supernovae, we obtain an upper limit on the lengthscale of the theory of $B_0 < 0.4$, or correspondingly $\lambda_1 < 1900\,\mathrm{Mpc}/h$ at 95% c.l. improving previous CMB constraints. For Yukawa–type models we get a bound on the coupling $0.75 < \beta_1 < 1.25$ at the 95% c.l. We also discuss the implications of the assumed priors on the estimation of modified gravity parameters, showing that a marginally less conservative choice improves the $f(R)$ constraints to $\lambda_1 < 1400\,\mathrm{Mpc}/h$, corresponding to $B_0 < 0.2$ at 95% c.l.

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I. INTRODUCTION

The observed acceleration of the cosmic expansion poses a challenge for modern cosmology. Standard general relativity (GR), applied to an expanding universe filled with radiation and cold dark matter, does not fit the data unless one invokes an additional component, either a cosmological constant, Λ, or a dynamical *dark energy* field [1]. The former case corresponds to the well known standard model of cosmology, or ΛCDM. An alternative approach to the phenomenon of cosmic acceleration consists of modifying the laws of gravity on large scales, in order to allow for self-accelerating solutions. Well-known examples of such theories are the $f(R)$ models [2–6], or more general scalar–tensor theories [7–10], the Dvali-Gabadadze-Porrati (DGP) model [11, 12], and its further extensions such as Degravitation [13].

Such theories need to fit the observed data as well as or better than the ΛCDM baseline model. This requirement is twofold: at the background level, the predicted expansion history has to be coherent with distance measurements such as those from Supernovae (SNe), the position of the CMB peaks, and baryon acoustic oscillations (BAO); on the other hand, the modified predictions for structure formation need to pass the test of comparison with the observed large scale structure (LSS) of the Universe. It has been shown for several cases — for instance for $f(R)$ [14–18] and DGP [19–24] — that the structure formation test can dramatically reduce the parameter space left unconstrained from the background test, and it is therefore instrumental to the quest for a truly viable model.

Structure formation in the Universe is driven by gravity through its potentials, which at the perturbative level in Newtonian gauge are described by two fields $\Psi(x_\mu)$ and $\Phi(x_\mu)$, defined as the perturbations to the time-time and space-space parts of the metric respectively. A powerful test of modified gravity is offered by measuring the past evolution of these potentials to compare it with the prediction of a given theory. In the standard ΛCDM model the potentials are expected to be identical, to remain constant during matter domination and to decay at late times, as a consequence of the ongoing transition to the dark energy phase. This is not necessarily the case for other theories. Metric perturbations are not viable astronomical observables, but we may reconstruct them from observational data which are directly dependent on them [25, 26]. The most useful data for this purpose are galaxy and cluster counts, weak lensing and the CMB.

The CMB is an almost perfectly isotropic black body radiation that has been generated at the epoch of hydrogen recombination and has undergone free streaming since then. Nonetheless, this free streaming may be altered if the CMB photons encounter potential wells which evolve in time. In this case, a non-zero net energy is gained (or lost) by the photons if the potential is becoming shallower (or deeper): this phenomenon, known as the integrated Sachs-Wolfe (ISW) effect [27], leaves an imprint on the larger scales of the CMB spectrum. In the standard ΛCDM theory, this effect is expected to be generated at late times as a consequence of the potential decay when the background starts accelerating. In models of modified gravity, such as $f(R)$ theories, the magnitude of this late effect can be altered, as well as potentials can evolve during matter domination, therefore generating an ISW effect at earlier redshifts [14, 16].

The ISW effect can be measured by cross-correlating the CMB with tracers of the large scale structure (LSS) [28], and it has been detected using several different data sets and was used to constrain cosmology [29–41]. The strongest ($4.5\,\sigma$) detection to date [42] has been obtained by combining multiple data sets at different redshifts, thus exploring the evolution of the potentials in time, and is fully consistent with the ΛCDM picture.

Here we focus on $f(R)$ theories and models with a Yukawa–type dark matter interaction, and use the ISW data by [42], in conjunction with the CMB, to test the general parametrisation of modified gravity by exploring the parameter space with a Monte Carlo Markov chain (MCMC) technique.

The plan of the paper is as follows. In Section II first we introduce the parametrisation used to describe departures from GR, and then specialise to the case of $f(R)$ and Yukawa–type theories. Then we review the method of the analysis and the data used in Section III. In Section IV we give details of our results and in Section V we discuss the effects of the assumed priors on the parameter estimation, before concluding in Section VI.

II. PARAMETRISED MODIFIED GRAVITY

Here we describe the formalism we use to parametrise departures from general relativity.

A. Background expansion

In our analysis we fix the background to that of the ΛCDM model of cosmology. The reasons for this choice are multiple; ΛCDM is currently the best fit to available data and popular models of modified gravity, e.g. $f(R)$, closely mimic ΛCDM at the background level with differences which are typically smaller than the precision achievable with geometric tests [43]. The most significant departures happen at the level of growth of structure and, by restricting ourselves to ΛCDM backgrounds, we can isolate them.

B. Structure formation

In models of modified gravity we expect departures from the standard growth of structure, even when the expansion history matches exactly the ΛCDM one. In general, the rate of clustering of dark matter, as well as the evolution of the metric potentials, is changed and can be scale-dependent. Moreover, typically there might be an effective anisotropic stress introduced by the modifications and the two potentials, Φ and Ψ, are not necessarily equal, as it was the case for ΛCDM [14, 16, 17, 19–22, 44]. Here we focus on the effect of the modified evolution of the potential, $\Phi+\Psi$, on the CMB. In order to evolve the potentials we employ the MGCAMB code developed in [18] (and publicly available at `http://www.sfu.ca/~gza5/MGCAMB.html`) to evaluate the growth of perturbations in models of modified gravity. In this code, the energy-momentum equations remain the standard ones, and the modifications to the Poisson and anisotropy equations are encoded in two functions $\mu(a,k)$ and $\gamma(a,k)$ defined by

$$k^2\Psi = -\frac{a^2}{2M_P^2}\mu(a,k)\rho\Delta\ , \tag{1}$$

$$\frac{\Phi}{\Psi} = \gamma(a,k)\,, \tag{2}$$

where $\rho\Delta \equiv \rho\delta + 3\frac{aH}{k}(\rho+P)v$ is the comoving density perturbation.

We will consider theories in which the modifications introduce an effective scalar degree of freedom (d.o.f.), and therefore a characteristic lengthscale. A typical action for such theories would be

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}g^{\mu\nu}\left(\nabla_\mu\phi\right)\left(\nabla_\nu\phi\right) - V(\phi)\right] + S_i\left(\chi_i, e^{-\alpha_i(\phi)/M_P}g_{\mu\nu}\right) \tag{3}$$

where ϕ represents the scalar d.o.f., χ_i is the $i^{\rm th}$ matter field and $\alpha_i(\phi)$ is the coupling of ϕ to χ_i. We will limit ourselves to cases in which the coupling is a linear function of the scalar field, i.e. $\alpha_i(\phi)\propto\phi$.

For the theories described by the action in Eq. (3), the functions μ and γ can be well represented by the following parametrisation introduced by [45] (and used in [18])

$$\mu(a,k) = \frac{1+\beta_1\lambda_1^2\,k^2a^s}{1+\lambda_1^2\,k^2a^s}\,, \tag{4}$$

$$\gamma(a,k) = \frac{1+\beta_2\lambda_2^2\,k^2a^s}{1+\lambda_2^2\,k^2a^s}\,, \tag{5}$$

where the parameters β_i can be thought of as dimensionless couplings, λ_i as dimensionful lengthscales and s is determined by the time evolution of the characteristic lengthscale of the theory, i.e. the mass of the scalar d.o.f. As shown in [18], in the case of scalar-tensor theories the parameters $\{\beta_i, \lambda_i^2\}$ are related in the following way

$$\beta_1 = \frac{\lambda_2^2}{\lambda_1^2} = 2-\beta_2\frac{\lambda_2^2}{\lambda_1^2} \tag{6}$$

and $1 \lesssim s \lesssim 4$.

1. $f(R)$ theories

These theories are a subclass of the models described by the action of Eq. (3), corresponding to the case of a universal fixed coupling $\alpha_i = \sqrt{2/3}\,\phi$, i.e. $\beta_1 = 4/3$ [46]. Moreover, $f(R)$ models that closely mimic ΛCDM correspond to $s\sim 4$ [18]. Therefore the number of free

parameters in Eqs. (4) and (5) can be reduced to one, e.g. the lengthscale λ_1.

The parametrisation in Eq. (4) effectively neglects a factor representing the rescaling of the Newton's constant already at the background level due to the modifications (e.g. $(1+f_R)^{-1}$ in $f(R)$ theories). Such a factor is very close to unity in models that satisfy local tests of gravity [43] and, as such, it can be neglected. However, when studying the $f(R)$ case, we need to include it to get a more precise estimate of the ISW effect for the MCMC analysis; therefore we use the following extension of Eq. (4)

$$\mu(a,k) = \frac{1}{1-1.4\cdot 10^{-8}|\lambda_1|^2 a^3}\,\frac{1+\frac{4}{3}\lambda_1^2\,k^2a^4}{1+\lambda_1^2\,k^2a^4}\,, \qquad (7)$$

where the difference with Eq. (4) is the multiplicative factor parametrising $(1+f_R)^{-1}$ which can be expressed in terms of λ_1 [43, 47]. Even with this extended parametrisation, we are left with a single free parameter, the lengthscale λ_1. This parameter can be easily related to the value of the mass scale of the scalar d.o.f. introduced by the addition of the $f(R)$ term to the Einstein-Hilbert action. This scalar d.o.f. is represented by the function $f_R \equiv df/dR$, also dubbed the *scalaron*, and λ_1 corresponds to its mass scale today, i.e. $\lambda_1 = 1/m^0_{f_R}$. In [14, 48] this family of $f(R)$ models was labelled by the parameter B_0 which is related to λ_1 via $\lambda_1^2 = B_0\,c^2/(2H_0^2)$. We will present our results mainly in terms of B_0 to facilitate the comparison.

2. *Yukawa–type dark matter interaction*

Scalar-tensor theories are an example of models with interaction between dark energy and dark matter. The parametrisation in Eqs. (7) and (5) is well suited also for models where dark matter (DM) particles self-interact *à la* Yukawa. Such models can be described by the presence of an additional mediator, represented by a scalar d.o.f., like the one in the action of Eq. (3), with a time-evolving mass and a coupling to DM particles which is now a free parameter of the model. In this case the only coupling of interest in Eq. (3) would be the one to DM, $\alpha_{\rm dm} \neq 0$. The coupling of this scalar d.o.f. to other matter fields, such as baryons, does not need to be the same of the one to DM; in particular, given the stringent constraints on such a coupling, it can be considered negligible.

In our parametrisation, the additional mediating force is characterised by the coupling β_1 (related to $\alpha_{\rm dm}$), the lengthscale λ_1 and its time evolution s. The remaining parameters, β_2 and λ_2, can be related to β_1 and λ_1 as in the scalar-tensor case of Eq. (6). In this case there is no need of any additional factor in the expression for μ, since it takes already into full account the effect of a Yukawa coupling on the clustering of dark matter, (in other words the Yukawa interaction alters the potential produced by a dark matter particle, but does not alter the background Newton's constant). Therefore, when analyzing models with Yukawa–type dark matter interaction we will use Eqs. (7) and (5) with free parameters $\{\beta_1, \lambda_1, s\}$.

Armed with the expressions for μ and γ to feed into MGCAMB, we can proceed to evaluate our observables.

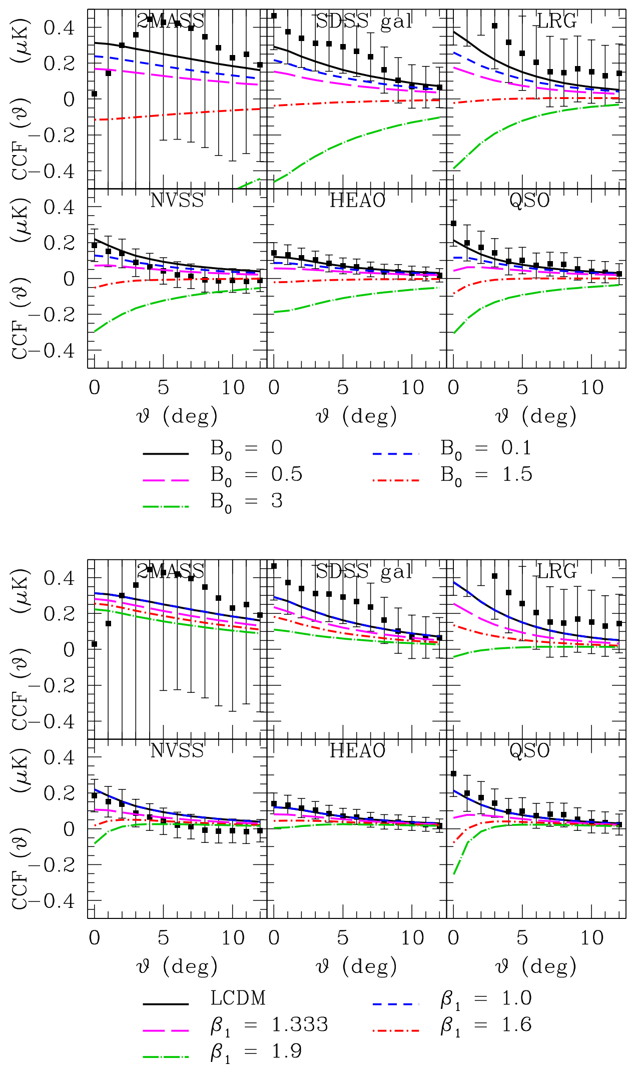

FIG. 1: Top: theoretical predictions for a family of $f(R)$ theories compared with our ISW data [42] measuring the angular CCF between the CMB and six galaxy catalogues. The model with $B_0 = 0$ is equivalent to ΛCDM , while increasing departures from GR produce negative cross-correlations. Bottom: the same for a family of Yukawa–like theories with fixed $B_0 = 2$. In this case non-unitary coupling generate a redshift evolution of the signal.

Parameter	Explanation	range (min, max)	
	Primary parameters	$f(R)$	Yukawa–type
ω_b	physical baryon density; $\omega_b = h^2\Omega_b$	$(0.005, 0.100)$	
ω_c	physical CDM density; $\omega_c = h^2\Omega_c$	$(0.01, 0.99)$	
ϑ_*	sound horizon angle; $\vartheta_* = 100 \cdot r_s(z_*)/D_A(z_*)$	$(0.5, 10.0)$	
τ	optical depth to reionisation	$(0.01, 0.80)$	
Ω_K	curvature density; $\Omega_K = 1 - \Omega_{\rm tot}$	0	
$\ln(10^{10}A_s^2)$	A_s is the scalar primordial amplitude at $k_{\rm pivot} = 0.05\,{\rm Mpc}^{-1}$	$(2.7, 4.0)$	
A_{SZ}	amplitude of the SZ template for WMAP and ACBAR	$(0, 2)$	
n_s	spectral index of primordial perturbations; $n_s - 1 = d\ln P/d\ln k$	$(0.5, 1.5)$	
B_0	present lengthscale of the theory (in units of the horizon scale)	$(0, 6)$	$(0, 6)$
β_1	coupling	$4/3$	$(0.001, 2)$
s	time evolution of the scalaron mass	4	$(1, 4)$
	Derived parameters		
H_0	Hubble parameter [km/s/Mpc]; calculated from ω_b, ω_c, ϑ, and Ω_K	tophat $(40, 100)$	
h	$h = H_0/(100\,{\rm km/s/Mpc})$	$(0.40, 1.00)$	
Ω_m	matter density parameter; $\Omega_m = (\omega_b + \omega_c)/h^2$		
Ω_Λ	vacuum energy density parameter; $\Omega_\Lambda = 1 - \Omega_K - \Omega_m$		
λ_1^2	lengthscale of the theory; $\lambda_1^2 = B_0 c^2/(2H_0^2)$	$(0, 5\cdot 10^7)$	$(0, 5\cdot 10^7)$
λ_2^2	second lengthscale of the theory; $\lambda_2^2 = \beta_1\lambda_1^2$	$(0, 7\cdot 10^7)$	$(0, 10^8)$
β_2	anisotropy parameter; $2/\beta_1 - 1$	$1/2$	$(0, 2000)$

TABLE I: Our primary MCMC sampling parameters and some important derived parameters. The modified gravity parameters are separated by a horizontal line.

III. ANALYSIS

A. Method

We run Monte Carlo Markov chains using the standard Cosmomc package [49], which is based on the CAMB CMB code [50] and is publicly available at `http://www.cosmologist.info/cosmomc`. As it is well known, this technique consists of using a Metropolis-Hastings algorithm to efficiently sample the multi-dimensional parameter space in which our model lives. The algorithm is based on multiple chains, which are started from different random initial points, and then evolved based on a prior probability distribution of the parameters, assumed flat in the chosen range. Each new step will be accepted if its likelihood improves the previous likelihood, weighted with some probability. The final posterior probability distribution is then directly obtained from the density distribution in the parameter space.

To study our modified gravity (MG) model we run five MCMC chains. We use the parameters described in Table I: this is based onto the usual set of ΛCDM parameters (note that the angle to the last scattering surface ϑ_* is used instead of the Hubble parameter H_0 since this reduces the degeneracies with the other parameters). To account for $f(R)$ theories we have to add one extra parameter: here we chose to use to use B_0 as a primary parameter. The other MG parameters are then fixed: $\lambda_1^2 = c^2B_0/(2H_0^2)$, $\lambda_2^2 = \beta_1\lambda_1^2$, $\beta_1 = 4/3$, $\beta_2 = 1/2$ and $s = 4$. In theories with Yukawa–type dark matter interaction we have two additional free parameters, i.e. the coupling β_1 and s as described above.

We shall discuss the effects of the parameter choice on the priors and on the results in more details in Section V.

B. Data

1. CMB

For the CMB we use the publicly released WMAP 5 years data [51] for the temperature and polarisation (TT, TE and EE) power spectra of the perturbations. In addition, we used flat top hat priors on the age of the Universe, $t_0 \in [10, 20]$Gyrs and on the Hubble parameter today $H_0 \in [40, 100]$km/s/Mpc. We include CMB lensing in the analysis.

2. ISW

To break the degeneracy which remains at the background level between GR and MG, we use the ISW data by [42]. These data were obtained by cross-correlating the WMAP maps of the CMB with six galaxy data sets in different bands (2MASS, SDSS main galaxies, LRGs and QSOs, NVSS, HEAO). The data span different redshift ranges from $\bar{z} = 0.1$ to $\bar{z} = 1.5$, thus allowing us to study the evolution of gravity, through the history of the decay of its potentials, in a tomographic way.

There are 78 actual data points, consisting of the angular cross-correlation functions (CCFs) in real space, binned at 12 angles between 0 and 12 deg for each of the catalogues. The full covariance matrix is highly non-diagonal both between the different angular bins and between the catalogues, due to overlaps in redshift and sky

coverage. For each MC model, the likelihood contribution due to the ISW data is calculated as follows: first the theoretical matter and matter-temperature power spectra C_l^{gg}, C_l^{Tg} are calculated through a full Boltzmann integration inside CAMB, then a Legendre transformation yields the CCFs and the matter auto-correlation functions (ACFs) at the relevant scales. Finally, the bias parameters, assumed constant for each catalogue, are recalculated for each model by forcing the ACFs to match the observations.

We show in Fig. 1 the CCF data points, overlapped with the theoretical predictions for a family of $f(R)$ and Yukawa–like theories. We can see that in the first case a departure from GR produces an unobserved negative signal, while in the second a coupling different from unity causes an equally unobserved redshift evolution of the effect.

3. *Supernovae*

To further constrain the background expansion history of the Universe, we study the effect of including the constraints from the Hubble diagram of distant Type Ia Supernovae (SNe). In particular we use the Union SN compilation by [52], which consists of 414 SNe drawn from 13 independent data sets plus 8 newly discovered SNe at low redshift, all reanalysed in a consistent way.

IV. RESULTS & CONSTRAINTS

A. $f(R)$ theories

In order to be conservative, we first perform an MCMC analysis using the constraints from CMB+ISW only, as it is in general good practice to add data sets gradually, to control that there are no strong tensions between them, which would produce artificially tight posteriors. We show in the left panel of Fig. 2 (red dashed lines) our constraints from this run. Here we see the one-dimensional marginalised likelihood distributions of the cosmological parameters. It can be seen that the usual ΛCDM parameters have likelihoods peaked around their standard values, while the extra parameter B_0 has an upper limit, so that we find

$$B_0 < 0.5 \quad \text{or} \quad \lambda_1 < 2000\,\text{Mpc}/h \ \ @\,95\%\,\text{c.l.} \tag{8}$$

We have checked the convergence of this and the following results from the $R-1$ statistic between the chains, requiring always $R-1 < 0.02$ and over 3000 estimated independent samples. This result, which has been made possible by a full combined CMB-ISW analysis data, improves the previous constraints of $B_0 < 1$ by [48], obtained only by considering models with a positive ISW-matter correlation signal.

We then perform a new analysis by adding the constraints from the Union supernovae catalogue by [52]. By adding these constraints, the parameter space allowed from the background expansion history becomes narrower, and thus so become the constraints on the modification of gravity, mainly due to the degeneracy of the theory wavelength λ_1 with other parameters such as Ω_m, as can be seen in the 2D contour plot of Fig. 3 (top panel). We can see the one-dimensional marginalised likelihood curves for the models in the left panel of Fig. 2 (black solid lines). The resulting constraint from the combination of CMB+ISW+SN is

$$B_0 < 0.4 \quad \text{or} \quad \lambda_1 < 1900\,\text{Mpc}/h \ \ @\,95\%\,\text{c.l.} \tag{9}$$

We have also tested how the results can improve by adding the latest additional priors on the value of the Hubble constant H_0 [53]; however, since the background is already well constrained by the intersection of CMB and SNe data, the improvement is marginal (at the percent level in the value of the upper limit).

B. Yukawa–type dark matter interaction

Finally we perform an analysis with three parameters to investigate theories in which dark matter particles interact via a Yukawa–type force. In this case, we directly include all data sets, i.e. CMB ISW and SNe, since we expect the marginalised constraints to be weaker due to the additional parameters. We can see the result of the 1D posterior likelihood in the right panel of Fig. 2. In this case, the upper limit on B_0 (or λ_1^2) is weakened and becomes uninteresting due to additional degeneracies, while we find a constraint on the coupling parameter

$$0.75 < \beta_1 < 1.25 \ \ @\,95\%\,\text{c.l.} \tag{10}$$

This limit is enhanced by the lack of strong redshift evolution of the ISW signal. The result is comparable with the previous constraints by [54], although obtained under different assumptions. We can not find any significant bound on the additional parameter s. We show in the bottom panels of Fig. 3 the 2D likelihood contours for Ω_m, B_0, Ω_m, β_1 and β_1, B_0.

From the last panel of Fig. 3 we can better understand why the constraint on B_0 is now weakened. In the $f(R)$ case, the coupling parameter is fixed to $\beta_1 = 4/3$, for which high values of B_0 are disfavoured. When the coupling is set free in the Yukawa-type theories, its preferred values are around $\beta_1 \simeq 1$, for which we can see that B_0 is unconstrained.

V. ON THE PARAMETRISATIONS AND THE BAYESIAN PRIORS

In Bayesian theory, the posterior probability of a model M with N paramters $\mathbf{\Theta}$ given the data $\mathbf{D}$ is obtained as

$$\mathcal{P}(\mathbf{\Theta}) = \frac{\mathcal{L}(\mathbf{\Theta})\Pi(\mathbf{\Theta})}{\mathcal{Z}(M)}, \tag{11}$$

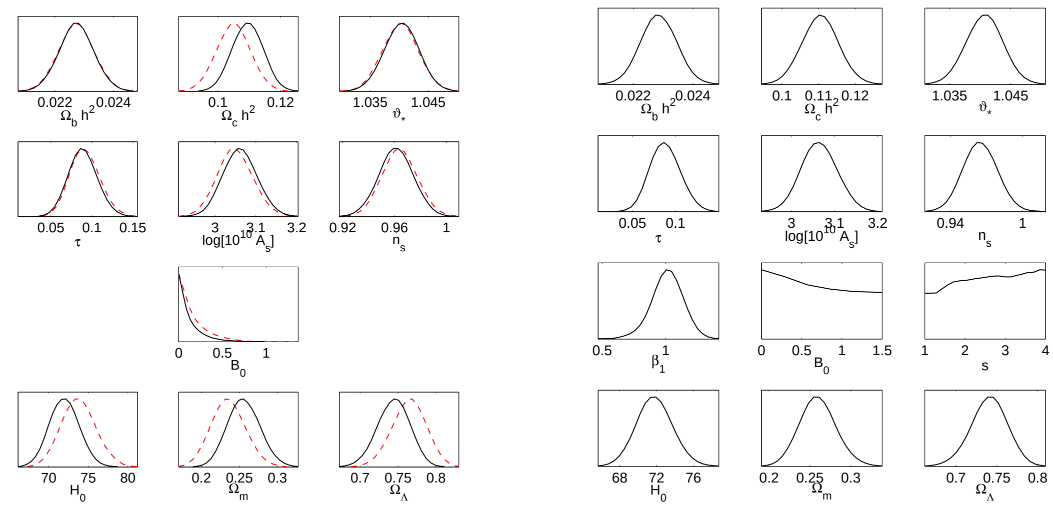

FIG. 2: Left: posterior likelihood distributions for the model parameters for the $f(R)$ case, using the combined CMB and ISW data (red, dashed lines) and adding also the SNe data (black, solid). We can see that the usual ΛCDM parameters peak around the concordance values, while the extra parameter B_0 has an upper limit. The SNe tighten the constraints by reducing the degeneracy between Ω_m and B_0. Right: the same for the Yukawa–type case. Here we only show the full CMB+ISW+SN result. The upper limit on B_0 disappears in this case due to the removal of the corrective factor from Eq. (7) and the additional degeneracies, and we can not constrain s either, but a value of the coupling β_1 close to unity is required to fit the data.

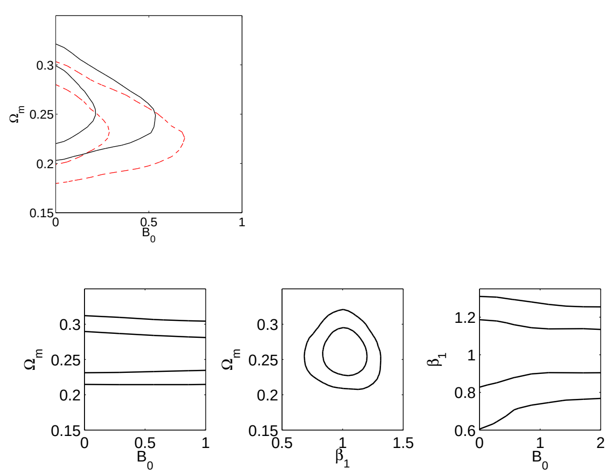

FIG. 3: 2D posterior likelihood distributions; the 68% and 95% probability contours are shown. In the top panel we can see the parameters Ω_m, B_0 for the $f(R)$ case, using the combined CMB and ISW data (red, dashed lines) and adding also the SNe data (black, solid). The SNe tighten the constraint by reducing the degeneracy between Ω_m and B_0. In the bottom panels, we show the Yukawa–type case for the parameters Ω_m, B_0 (left), Ω_m, β_1 (centre), and β_1, B_0 (right).

where $\mathcal{L}(\mathbf{\Theta})$ is the likelihood function, $\Pi(\mathbf{\Theta})$ the prior and $\mathcal{Z}(M)$ is the Bayesian evidence for the model, which we can here consider constant. The marginalised posterior of a single parameter ϑ is then obtained integrating over the other parameters:

$$\mathcal{P}(\vartheta) \propto \int \mathcal{L}(\mathbf{\Theta})\Pi(\mathbf{\Theta})d^{N-1}\mathbf{\Theta}. \qquad (12)$$

We normally assume the priors to be flat over the sampled interval, so that they can be ignored in the marginalisation process. In this case, the posterior is simply mapped by the likelihood, upon which assumption the MCMC technique is based. However, if we change one parameter as $\vartheta \to \zeta(\vartheta)$, we have that a prior which was flat in ϑ will not in general be flat in ζ. In other words, changing parametrisation modifies the prior from $\Pi(\vartheta)$ to

$$\Pi(\zeta) = \Pi(\vartheta)\frac{d\vartheta}{d\zeta}, \qquad (13)$$

which corresponds to a different weighting of the likelihood in Eq. (12); in higher dimensions, the derivative is replaced by the Jacobian of the transformation, as described for example in [55]. If we now run an MCMC chain using the new parameter ζ as a primary parameter, we will in practice force the prior $\Pi(\zeta)$ to be flat, meaning that the prior on the old parameter $\Pi(\vartheta)$ will be tilted by a factor $d\zeta/d\vartheta$. If a prior is tilted, it will cause a downweighting of the parameter region in one direction, thus affecting the results. All this is well known, and it has been discussed by [49].

Clearly, if the theory is well constrained by the data, then the interval of integration will be small, and the prior will be approximately flat in any parametrisation, yielding consistent posteriors. However, this does not apply to our case: since the current data are not yet strongly constraining MG, we have found that the choice of priors does indeed have strong effects. In this case, we read in [49] that

> "...for the results of the parameter estimation to be meaningful it is essential that the priors

on the base set are well justified..."

This leaves us with the problem of deciding which of the many possible MG parameters $(B_0, \lambda_1, \lambda_1^2, \mathrm{Log}\lambda_1^2, ...)$ is most *meaningful* or *well justified* to be assumed having flat priors. The conclusion seems uncertain since any modification of gravity is currently based on speculation without experimental backing, and it is thus hard to decide which parametrisation ought to be preferred on physical grounds. Since there is no evidence for a modification of GR, we have then decided to make the most conservative choice by presenting the parametrisation which yields the weakest constraints on this modification.

Nevertheless, at least in the simplest $f(R)$ case with one extra parameter, we can forecast the effect of the different options by using Eq. (13). If we take the parameter λ_1 as our baseline choice, we can predict that by switching to flat priors in the other parameters we will have the effects summarised in Table II: any transformation to a parameter ζ with $d^2\zeta/d(\lambda_1)^2 > 0$ will tilt the priors — and bias the posteriors — towards higher values of λ_1; the opposite for $d^2\zeta/d(\lambda_1)^2 < 0$.

Finally we compare these predictions with the results we infer on the baseline parameter λ_1 by running MCMC chains using different primary parameters; we find a good agreement with the prediction, and in particular we confirm that our choice of B_0 (or equivalently λ_1^2) is the most conservative one. If we decide that the wavelength λ_1 should be considered the most physically justified quantity, we then obtain the following stricter bounds in the $f(R)$ case using CMB+SN+ISW:

$$\lambda_1 < 1400\,\mathrm{Mpc}/h \quad \text{or} \quad B_0 < 0.2 \;\; @\,95\%\,\mathrm{c.l.} \tag{14}$$

VI. CONCLUSIONS

In this paper we have presented an analysis of parametrised modified gravity theories in light of recent data from the CMB and the large scale structure of the Universe.

We have focused first on the $f(R)$ class of modified gravities, which are described by a single extra parameter related to the mass of the scalaron, which can be parametrised as its present wavelength B_0. We have performed a full MCMC analysis and found some new constraints on this parameter: in the $f(R)$ case, we have obtained $B_0 < 0.4$ or $\lambda_1 < 1900\,\mathrm{Mpc}/h$ at the 95% c.l. when using combined data from the ISW, the CMB and SNe. This puts strong bounds on the parameter space of such a theory in light of current data, therefore further reducing the possibilities for significant infrared modifications of general relativity.

We have highlighted how the choice of the parametrisation, through its effect on the Bayesian priors, can be a very sensitive issue in the estimation of non-standard cosmological parameters. We have shown that by using a marginally less conservative prior we can reduce the posterior constraint to $\lambda_1 < 1400\,\mathrm{Mpc}/h$ or $B_0 < 0.2$ at the 95% c.l.

We have then studied theories with a Yukawa–type interaction between dark matter particles; these models require two further parameters: the coupling β_1 and the time evolution of the scalaron mass s. In this case we have shown how the constraints on the theory lengthscale weakens, but we obtain a bound on the coupling $0.75 < \beta_1 < 1.25$ at the 95% c.l. when using all the data sets.

Our result proves once more that tests of structure formation and attempts to reconstruct the evolution of the gravitational potentials are crucial to distinguish between modifications of gravity and dark energy theories. Future improvements along these lines will be likely achieved by combining all the different available probes, such as the ISW, weak lensing, galaxy cluster counts and peculiar velocities (see for instance [18, 56–59] for forecasts) [61].

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parameter ζ	$d\zeta/d\lambda_1$	$d^2\zeta/d(\lambda_1)^2$	prior on λ_1	expected posterior on λ_1	MCMC estimated λ_1 @ 95% c.l.
λ_1	1	0	flat	baseline	$\lambda_1 < 1400$ Mpc/h
λ_1^2	$\propto \lambda_1$	> 0	positive tilt	biased to higher λ_1	$\lambda_1 < 1900$ Mpc/h
B_0	$\propto \lambda_1$	> 0	positive tilt	biased to higher λ_1	$\lambda_1 < 1900$ Mpc/h
Log λ_1^2	$\propto 1/\lambda_1$	< 0	negative tilt	biased to lower λ_1	$\lambda_1 < 700$ Mpc/h

TABLE II: Effect of the priors on the estimation of the baseline parameter λ_1. Using flat priors on different parameters ζ defined by transformations whose second derivative is positive (negative) will produce effective tilted priors on λ_1. This will overestimate regions with higher (lower) λ_1, thus leading to biased posteriors. The results of MCMC chains using different parameters agree with this prediction.

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[61] During the preparation of this paper, a new constraint on $f(R)$ theories came from [60], based on local clusters abundance that improves previous constraints in [48] by two orders of magnitude. The new constraint presented here is compatible with this constraint, involves only linear perturbation theory and it is therefore fully independent and complementary.